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The reconstruction of local quantum operators for the boundary *XXZ* spin-$\frac{1}{2}$ Heisenberg chain

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Abstract. The method of the reconstruction of local quantum operators in terms of the elements of the quantum monodromy matrix is applied to the *XXZ* spin-$\frac{1}{2}$ Heisenberg chain with integrable boundary conditions.

1. Introduction

In the theory of quantum integrable models, the calculation of the spectrum and the correlation functions are two main problems. The spectrum of the integrable model may be obtained by the general method called the Bethe Ansatz and quantum inverse scattering method (QISM) [1–4]. But the calculation of the correlation functions is a very difficult problem in general situations. For example, in the case of the Heisenberg spin chains [5], whose spectrum was solved early [1, 6, 7], the first manageable expressions for its correlation functions were only given in 1992, and later by Jimbo *et al* [8, 9], on the basis of the symmetry structure of the infinite chain. The differential equations and determinant representation for its correlation functions were given in 1994 and later by Korepin *et al* [10–12] based on the algebra Bethe Ansatz method. Recently, the explicit reconstruction of any local spin operator at any site of the chain in terms of the elements of the quantum monodromy matrix was solved for this model by Kitanine *et al* [13]. The explicit solution has also been obtained for a large class of lattice quantum integrable models [14, 15]. This put forward an approach to calculate the correlation functions. It is well known that in the frame of QISM, the Bethe eigenstates are created by the successive action of the elements of the monodromy matrix $B(\lambda_k)$ (or $C(\lambda_k)$), which are non-local operators. So in the calculation of correlation functions, one has to deal with two types of operators: local operators σ_n^i and non-local operators $B(\lambda_k)$ (or $C(\lambda_k)$). The commutation relations between these two types of operators are generally complicated. This makes it difficult to calculate the correlation functions. Once the local operators are reconstructed in the terms of creation and annihilation operators, one only needs to deal with non-local operators, whose commutation relations obey the well known Yang–Baxter equation.

On the other hand, since the systematical treatment of the independent boundary conditions for integrable quantum systems was proposed by Sklyanin [16], many integrable models have been discussed for the case of open boundary conditions [17–21]. When the boundary conditions on the finite interval are compatible with integrability, the monodromy matrix can be constructed and the algebra Bethe Ansatz can be applied to diagonalizing the quantum

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monodromy matrix. In computing the correlation function, any local operator at any site is also expected to be reconstructed from the elements of the monodromy matrix. In this paper, we apply the method proposed in [13–15] to the partially anisotropic *XXZ* spin-$\frac{1}{2}$ chain with integrable boundary.

The paper is organized as follows. In section 2, the main features of the boundary *XXZ* spin-$\frac{1}{2}$ model are listed and the necessary notation is introduced. In section 3, the method of reconstructing local quantum operators is applied to this model and the main result (26) is obtained. In the last section, some possible generalizations to other lattice quantum integrable models are discussed.

2. Boundary *XXZ* spin-$\frac{1}{2}$ chain

In this section, let us briefly recall some of the main results of the boundary *XXZ* spin-$\frac{1}{2}$ chain [16], which will be used in the next section.

The Hamiltonian of the model reads

$$H = \sum_{n=1}^{N-1}(\sigma_n^1\sigma_{n+1}^1 + \sigma_n^2\sigma_{n+1}^2 + \cosh\eta\cdot\sigma_n^3\sigma_{n+1}^3) + \sinh\eta\cdot(\coth\xi_-\cdot\sigma_1^3 + \coth\xi_+\cdot\sigma_N^3). \tag{1}$$

Here, $\sigma_n^i = \overbrace{\cdots 1\otimes \underset{n\text{th}}{\sigma^i}\otimes 1\otimes\cdots}^{N}$, $\sigma^i (i = 1, 2, 3)$, are Pauli matrices. The R matrix, which plays an important role in QISM, is given by

$$R(u) = \begin{pmatrix} a(u) & & & \\ & b(u) & c(u) & \\ & c(u) & b(u) & \\ & & & a(u) \end{pmatrix} \tag{2}$$

where $a(u) = \sinh(u+\eta), b(u) = \sinh(u), c(u) = \sinh(\eta)$. The R matrix (2) satisfies the Yang–Baxter equation,

$$R_{12}(u_1-u_2)\cdot R_{13}(u_1-u_3)\cdot R_{23}(u_2-u_3) = R_{23}(u_2-u_3)\cdot R_{13}(u_1-u_3)\cdot R_{12}(u_1-u_2) \tag{3}$$

the conditions of unitarity

$$R_{12}(u)\cdot R_{12}(-u) = \rho(u) = -\sinh(u+\eta)\sinh(u-\eta) \tag{4}$$

and crossing unitarity

$$R_{12}^{t_1}(u)\cdot R_{12}^{t_1}(-u-2\eta) = \tilde{\rho}(u) = \rho(u+\eta). \tag{5}$$

In the case of bulk, define

$$T_{i_1i_2\ldots i_N}(u) = L_{i_N}(u)\ldots L_{i_2}(u)L_{i_1}(u). \tag{6}$$

The monodromy matrix is constructed as $T(u) = T_{12\ldots N}(u)$. $L_n(u)$ are some representations of the associative algebra connected with (2):

$$R_{12}(u_1-u_2)L_n^{(1)}(u_1)L_n^{(2)}(u_2) = L_n^{(2)}(u_2)L_n^{(1)}(u_1)R_{12}(u_1-u_2). \tag{7}$$

Here, $L_n^{(1)}(u) = L_n(u)\otimes 1$, $L_n^{(2)}(u) = 1\otimes L_n(u)$, and $L_n(u)$ act on the space $V\otimes W_n$. The V is auxiliary space C^2 and W_n the quantum spaces at the site n. Due to (6), (7), $T_{i_1i_2\ldots i_N}(u)$ satisfy the same commutation relation as $L_n(u)$ do (7). When W_i are isomorphic to V, $L_n(u)$ may be realized as a 2×2 matrix in auxiliary space V,

$$L_n(u) \equiv R_{0n}(u-u_n-\tfrac{1}{2}\eta) = \begin{pmatrix} L_n^{11}(u) & L_n^{12}(u) \\ L_n^{21}(u) & L_n^{22}(u) \end{pmatrix} \tag{8}$$

and the elements L_n^{ij} read as

$$\begin{aligned} L_n^{11} &= \sinh(u-u_n)\coth\left(\frac{\eta}{2}\right) + \coth(u-u_n)\sinh\left(\frac{\eta}{2}\right)\sigma_n^3 \\ L_n^{12} &= \sigma_n^- \sinh(\eta) \qquad L_n^{21} = \sigma_n^+ \sinh(\eta) \\ L_n^{22} &= \sinh(u-u_n)\coth\left(\frac{\eta}{2}\right) - \coth(u-u_n)\sinh\left(\frac{\eta}{2}\right)\sigma_n^3 \end{aligned} \tag{9}$$

where $\sigma_n^{\pm} = \frac{1}{2}(\sigma_n^1 \pm i\sigma_n^2)$, and u_n is the fixed parameter dependent on the site n. For the *XXZ* spin-$\frac{1}{2}$ chain, all parameters u_n are equal to $-\frac{1}{2}\eta$ (homogeneous case). In this paper, all discussions relate to the inhomogeneous case; but then letting $u_n = -\frac{1}{2}\eta$ one obtains the results for the homogeneous case. Similarly, $T(u)$ may be also expressed as a 2×2 matrix in auxiliary space V,

$$T(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}. \tag{10}$$

A, B, C and D are linear operators acting on quantum space $\prod_{i=1}^N \otimes W_i$. The transfer matrix is defined as

$$t(u) = \mathrm{tr}_0\, T(u) = A(u) + D(u)$$

where, tr_0 means the trace in auxiliary space. In the frame of QISM, diagonalizing the Hamiltonian is equivalent to calculating the eigenvalue of the transfer matrix and constructing the eigenvectors $\prod_i B(v_i)w_+$. Here v_i must satisfy the set of equations called Bethe equations, and the highest vector w_+ satisfy

$$A(u)w_+ = \delta_+(u)w_+ \qquad D(u)w_+ = \delta_-(u)w_+ \qquad C(u)w_+ = 0 \tag{11}$$

where $\delta_+(u) = \prod_{n=1}^N \sinh(u-u_n+\frac{1}{2}\eta)$, $\delta_-(u) = \prod_{n=1}^N \sinh(u-u_n-\frac{1}{2}\eta)$. When $u = u_n + \frac{1}{2}\eta$, from (8) one knows that $L_n(u)$ reduce to $\sinh\eta\cdot P_{0n}$, and $t(u_n+\frac{1}{2}\eta)$ is an invertible shift operator acting on $T_{i_1 i_2 \ldots i_N}(u)$ as [13, 14]

$$t(u_n + \tfrac{1}{2}\eta)\cdot T_{n\ldots N1\ldots n-1}(u) = T_{n+1\ldots N1\ldots n}(u)\cdot t(u_n+\tfrac{1}{2}\eta). \tag{12}$$

In the case of integrable boundary conditions, besides R matrix (2), the boundary scattering matrices $K_{\pm}(u)$ are necessary to keep the integrability of the model,

$$\begin{aligned} &K_+(u) = K(u+\tfrac{1}{2}\eta, \xi_+) \qquad K_-(u) = K(u-\tfrac{1}{2}\eta, \xi_-) \\ &K(u,\xi) = \begin{pmatrix} \sinh(u+\xi) & \\ & -\sinh(u-\xi) \end{pmatrix}. \end{aligned} \tag{13}$$

$K_{\pm}(u)$ satisfy the boundary Yang–Baxter equations [16],

$$\begin{aligned} &R_{12}(u_1-u_2)\cdot K_-^1(u_1)\cdot R_{12}(u_1+u_2-\eta)\cdot K_-^2(u_2) \\ &\quad = K_-^2(u_2)\cdot R_{12}(u_1+u_2-\eta)\cdot K_-^1(u_1)\cdot R_{12}(u_1-u_2) \end{aligned} \tag{14}$$

$$\begin{aligned} &R_{12}(-u_1+u_2)\cdot K_+^1(u_1)^{t_1}\cdot R_{12}(-u_1-u_2-\eta)\cdot K_+^2(u_2)^{t_2} \\ &\quad = K_+^2(u_2)^{t_2}\cdot R_{12}(-u_1-u_2-\eta)\cdot K_+^1(u_1)^{t_1}\cdot R_{12}(-u_1+u_2) \end{aligned} \tag{15}$$

$K_{\pm}^1(u_1) = K_{\pm}(u_1)\otimes 1$ and $K_{\pm}^2(u_2) = 1\otimes K_{\pm}(u_2)$. The monodromy matrix is defined as [16]

$$U^t(u) = T^t(u)\cdot K_+^t(u)\cdot\sigma^2\cdot T(-u)\cdot\sigma^2 = \begin{pmatrix} \mathcal{A}(u) & \mathcal{C}(u) \\ \mathcal{B}(u) & \mathcal{D}(u) \end{pmatrix}. \tag{16}$$

Utilizing (7), (15), $U(u)$ can be proved to satisfy (15) as $K_+(u)$ do. This results in the commutation relations between $\mathcal{A}(u)$, $\mathcal{B}(u)$, $\mathcal{C}(u)$ and $\mathcal{D}(u)$. The transfer matrix is defined as

$$\begin{aligned} \tau(u) &= \mathrm{tr}_0\, U(u)\cdot K_-(u) \\ &= \sinh(u-\tfrac{1}{2}\eta+\xi_-)\mathcal{A}(u) - \sinh(u-\tfrac{1}{2}\eta-\xi_-)\mathcal{D}(u). \end{aligned} \tag{17}$$

The eigenvectors are constructed as

$$|v_1 \dots v_M\rangle = \mathcal{B}(v_1)\dots\mathcal{B}(v_M)w_+. \tag{18}$$

v_i satisfy a set of equations called Bethe equations. The highest vectors of $T(u)$, w_+ are also those of $U(u)$:

$$\mathcal{A}(u)w_+ = \Delta_+(u)w_+ \qquad \mathcal{D}(u)w_+ = \Delta_-(u)w_+ \qquad \mathcal{C}(u)w_+ = 0. \tag{19}$$

Due to (10), (13) and (16), the terms of the matrix $U(u)$ can be written as

$$\begin{aligned}
\mathcal{A}(u) &= \sinh(u + \tfrac{1}{2}\eta + \xi_+)A(u)D(-u) + \sinh(u + \tfrac{1}{2}\eta - \xi_+)C(u)B(-u)\\
\mathcal{B}(u) &= \sinh(u + \tfrac{1}{2}\eta + \xi_+)B(u)D(-u) + \sinh(u + \tfrac{1}{2}\eta - \xi_+)D(u)B(-u)\\
\mathcal{C}(u) &= -\sinh(u + \tfrac{1}{2}\eta + \xi_+)A(u)C(-u) - \sinh(u + \tfrac{1}{2}\eta - \xi_+)C(u)A(-u)\\
\mathcal{D}(u) &= -\sinh(u + \tfrac{1}{2}\eta + \xi_+)B(u)C(-u) - \sinh(u + \tfrac{1}{2}\eta - \xi_+)D(u)A(-u).
\end{aligned} \tag{20}$$

The eigenvalues of $\mathcal{A}(u)$ and $\mathcal{D}(u)$ can be expressed in those of $A(u)$ and $D(u)$ as

$$\begin{aligned}
\Delta_+(u) = \frac{1}{\sinh(2u)}\{&\sinh(2u+\eta)\sinh(u - \tfrac{1}{2}\eta + \xi_+)\delta_+(u)\delta_-(-u)\\
&+ \sinh(\eta)\sinh(u + \tfrac{1}{2}\eta - \xi_+)\delta_+(-u)\delta_-(u)\}
\end{aligned} \tag{21}$$

$$\Delta_-(u) = -\sinh(u + \tfrac{1}{2}\eta - \xi_+)\delta_+(-u)\delta_-(u). \tag{22}$$

3. Reconstruction of local quantum operators

Here reconstruction of quantum operators means reconstructing the local spin operators $\sigma_n^\pm$ and σ_n^z at a given site n of the chain in terms of the matrix elements $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ of the monodromy matrix $U(u)$. Instead of $\sigma_n^\pm$ and σ_n^z, in the following we will consider the general operators E_n^{ij}, $i, j \in \{1, 2\}$, acting on the local quantum space $W_n \simeq C^2$ at site n as the 2×2 matrix, $(E_n^{ij})_{kl} = \delta_k^i\delta_l^j$, $k, l \in \{1, 2\}$.

Firstly, let us define

$$U^t_{i_1\dots i_N}(u) = T^t_{i_1\dots i_N}(u)\cdot K^t_+(u)\cdot\sigma^2\cdot T_{i_1\dots i_N}(-u)\cdot\sigma^2. \tag{23}$$

It is obvious that $U_{12\dots N}(u) = U(u)$. Owing to the fact that $t(u_n + \frac{1}{2}\eta)$ is an invertible shift operator acting on quantum space $\prod_{i=1}^N \otimes W_i$, one can prove that $t(u_n + \frac{1}{2}\eta)$ act on $U^t_{n\dots N1\dots n-1}(u)$ as

$$\begin{aligned}
&t(u_n + \tfrac{1}{2}\eta)\cdot U^t_{n\dots N1\dots n-1}(u)\\
&\qquad = t(u_n + \tfrac{1}{2}\eta)\cdot T^t_{n\dots N1\dots n-1}(u)\cdot K^t_+(u)\cdot\sigma^2\cdot T_{n\dots N1\dots n-1}(-u)\cdot\sigma^2.
\end{aligned} \tag{24}$$

Noting that superscript t means transpose in auxiliary space and using equation (12), one has

$$\begin{aligned}
(24) &= [t(u_n + \tfrac{1}{2}\eta)\cdot T_{n\dots N1\dots n-1}(u)]^t\cdot K^t_+(u)\cdot\sigma^2\cdot T_{n\dots N1\dots n-1}(-u)\cdot\sigma^2\\
&= T^t_{n+1\dots N1\dots n}(u)\cdot K^t_+(u)\cdot\sigma^2\cdot t(u_n + \tfrac{1}{2}\eta)\cdot T_{n\dots N1\dots n-1}(-u)\cdot\sigma^2\\
&= T^t_{n+1\dots N1\dots n}(u)\cdot K^t_+(u)\cdot\sigma^2\cdot T_{n+1\dots N1\dots n}(-u)\cdot\sigma^2\cdot t(u_n + \tfrac{1}{2}\eta)\\
&= U^t_{n+1\dots N1\dots n}(u)\cdot t(u_n + \tfrac{1}{2}\eta).
\end{aligned} \tag{25}$$

So, $\prod_{i=1}^{n-1} t(u_i + \frac{1}{2}\eta)\cdot U^t(u) = U^t_{n\dots N1\dots n-1}(u)\cdot\prod_{i=1}^{n-1} t(u_i + \frac{1}{2}\eta)$. Then one has the following proposition.

Proposition 1. *An operator $E_n^{ij} \in End(W_n)$ in a given site n of the chain can be expressed in the following way:*

$$E_n^{ij} = \prod_{i=1}^{n-1} t(u_i + \tfrac{1}{2}\eta) \cdot \mathrm{tr}_0[U(u_n + \tfrac{1}{2}\eta) \cdot E_0^{ij} \cdot K_-(u_n + \tfrac{1}{2}\eta)]\tau(u_n + \tfrac{1}{2}\eta)^{-1}\left(\prod_{i=1}^{n-1} t(u_i + \tfrac{1}{2}\eta)\right)^{-1} \tag{26}$$

where E_0^{ij} are matrices acting on auxiliary space as E_n^{ij} on n-site quantum space.

Proof. Because E_0^{ij} commute with $t(u_i + \frac{1}{2}\eta)$,

$$\begin{aligned}
&\prod_{i=1}^{n-1} t(u_i + \tfrac{1}{2}\eta) \cdot \mathrm{tr}_0[U(u_n + \tfrac{1}{2}\eta) \cdot E_0^{ij} \cdot K_-(u_n + \tfrac{1}{2}\eta)] \\
&\qquad = \prod_{i=1}^{n-1} t(u_i + \tfrac{1}{2}\eta) \cdot \mathrm{tr}_0[E_0^{ji} \cdot U^t(u_n + \tfrac{1}{2}\eta) \cdot K_-^t(u_n + \tfrac{1}{2}\eta)] \\
&\qquad = \mathrm{tr}_0\left[E_0^{ji} \cdot \prod_{i=1}^{n-1} t(u_i + \tfrac{1}{2}\eta) \cdot U^t(u_n + \tfrac{1}{2}\eta) \cdot K_-^t(u_n + \tfrac{1}{2}\eta)\right] \\
&\qquad = \mathrm{tr}_0\left[E_0^{ji} \cdot U^t_{n\ldots N1\ldots n-1}(u_n + \tfrac{1}{2}\eta) \cdot \prod_{i=1}^{n-1} t(u_i + \tfrac{1}{2}\eta) \cdot K_-^t(u_n + \tfrac{1}{2}\eta)\right].
\end{aligned} \tag{27}$$

Note that $L_n^t(u_n + \frac{1}{2}\eta) = \sinh\eta \cdot P_{0n}^t$, which act on E_0^{ji} as

$$E_0^{ji} \cdot L_n^t(u_n + \tfrac{1}{2}\eta) = \sinh\eta \cdot (P_{0n} \cdot E_0^{ij})^t = \sinh\eta \cdot (E_n^{ij} \cdot P_{0n})^t = E_n^{ij} \cdot L_n^t(u_n + \tfrac{1}{2}\eta). \tag{28}$$

(6), (23) and (28) result in

$$E_0^{ji} \cdot U^t_{n\ldots N1\ldots n-1}(u_n + \tfrac{1}{2}\eta) = E_n^{ij} \cdot U^t_{n\ldots N1\ldots n-1}(u_n + \tfrac{1}{2}\eta). \tag{29}$$

Owing to (28), (29), one has

$$\begin{aligned}
(27) &= \mathrm{tr}_0\left[E_n^{ij} \cdot U^t_{n\ldots N1\ldots n-1}(u_n + \tfrac{1}{2}\eta) \cdot \prod_{i=1}^{n-1} t(u_i + \tfrac{1}{2}\eta) K_-^t(u_n + \tfrac{1}{2}\eta)\right] \\
&= E_n^{ij} \cdot \mathrm{tr}_0\left[U^t_{n\ldots N1\ldots n-1}(u_n + \tfrac{1}{2}\eta) \cdot \prod_{i=1}^{n-1} t(u_i + \tfrac{1}{2}\eta) K_-^t(u_n + \tfrac{1}{2}\eta)\right] \\
&= E_n^{ij} \cdot \mathrm{tr}_0\left[\prod_{i=1}^{n-1} t(u_i + \tfrac{1}{2}\eta) \cdot U^t(u_n + \tfrac{1}{2}\eta) \cdot K_-^t(u_n + \tfrac{1}{2}\eta)\right] \\
&= E_n^{ij} \cdot \prod_{i=1}^{n-1} t(u_i + \tfrac{1}{2}\eta) \cdot \tau(u_n + \tfrac{1}{2}\eta).
\end{aligned} \tag{30}$$

Multiply both sides of (30) by $(\prod_{i=1}^{n-1} t(u_i + \frac{1}{2}\eta) \cdot \tau(u_n + \frac{1}{2}\eta))^{-1}$ from the right, and one obtains the final result. In the homogeneous case, all spectral parameters u_n limit to $-\frac{1}{2}\eta$, we have the following explicit expressions:

$$\begin{aligned}
\sigma_n^z &= [A(0) + D(0)]^{n-1}[\sinh(\xi_- - \tfrac{1}{2}\eta)\mathcal{A}(0) - \sinh(\xi_- + \tfrac{1}{2}\eta)\mathcal{D}(0)] \\
&\qquad \times[\mathcal{A}(0) + \mathcal{D}(0)]^{-1}[A(0) + D(0)]^{-n+1} \\
\sigma_n^+ &= [A(0) + D(0)]^{n-1}[\sinh(\xi_- + \tfrac{1}{2}\eta)\mathcal{C}(0)][\mathcal{A}(0) + \mathcal{D}(0)]^{-1}[A(0) + D(0)]^{-n+1} \\
\sigma_n^- &= [A(0) + D(0)]^{n-1}[\sinh(\xi_- - \tfrac{1}{2}\eta)\mathcal{B}(0)][\mathcal{A}(0) + \mathcal{D}(0)]^{-1}[A(0) + D(0)]^{-n+1}.
\end{aligned} \tag{31}$$

□

4. Discussion

In this paper, we have reconstructed the local spin operators at any site in terms of elements of the monodromy matrix for the boundary *XXZ* spin-$\frac{1}{2}$ model, and obtained the main result (26). Here, the basis of derivation is that the quantum Lax operator $L_n(u)$ reduces to the permutation operators P_{on} when $u = u_n + \frac{1}{2}\eta$. Hence, as in the case of periodic boundary conditions, the result may be applied to other lattice models and fused lattice models with integrable boundary conditions as has been done for periodic boundary conditions. In essence, there is no difficulty in dealing with these models, except that fusion in auxiliary space is necessary to make the auxiliary space isomorphic to local quantum space. Finally, it is interesting to compare our main results (26) with the analogous formulae in [13–15]. Because the shift operator is constructed from the transfer matrix, the right-hand side of the formula in [13–15] only involves the elements of the monodromy matrix. So it may be used immediately in calculating the correlation function. But in the case of the boundary, the shift operator is constructed from the bulk transfer matrix. Two types of operators, $A(D)$ and $\mathcal{A}(\mathcal{B}, \mathcal{C}, \mathcal{D})$, appear in (26). $\mathcal{A}(\mathcal{B}, \mathcal{C}, \mathcal{D})$ are the elements of the monodromy matrix, and $A(D)$ are the elements of the bulk monodromy matrix. Although these two types of operators have the common highest vector w_+, the algebraic Bethe Ansatz states of the open chain are not eigenfunction of the shift operators $(A + D)$. This makes it difficult to calculate the correlation function using (26) immediately. One must know how to express the elements of the shift operator in that of the monodromy matrix. It appears difficult to solve this problem, although one can express the elements of the monodromy matrix in that of the bulk monodromy matrix from (20).

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